

Oscillations of 2D Rashba system spin polarization in quantizing magnetic field

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INTRODUCTION

There are two main types of spin-orbit coupling in quantum wells based on semiconductors having the Zincblende structure: Rashba interaction [1], caused by the structural asymmetry of the quantum well, and Dresselhaus interaction [2], originating due to the structural inversion asymmetry of the bulk material. In the approximation linear in the electron momentum, both contributions can be formally represented in ways similar to each other. The spin-orbit interaction (SOI) leads to correlation of the translational and spin motion of electrons. Such correlation is the origin of many transport phenomena observed in such systems. Among such phenomena, there are beats in Shubnikov—de Haas oscillations [3], spin accumulation [4], magneto-electric effect [5], etc. In the cases when the SOI is in some sense small, one can perform a canonical transformation of the Hamiltonian, that decouples kinetic and spin degrees of freedom. All other terms in the Hamiltonian, describing the interaction of electrons with the lattice and external fields (if any) also undergo the transformation. In the latter case, the effective interaction of electrons in the system with external fields appears, which leads to resonant absorption of the field energy not only at the frequency of the paramagnetic resonance ω_s or cyclotron resonance ω_c , but also at their linear combinations [6]. A gauge-invariant theory of this phenomenon has been developed in [7]. This interaction also determines the gauge-invariant equations of motion for macroscopic variables characterizing the system. We apply the method developed in [7] in order to describe the kinetics of 2D conductivity electrons in electric and quantizing magnetic fields when the spin-orbit interaction is present. We write down and solve the macroscopic equations for the energy balance of the spin subsystem and for transverse spin components; consider the magneto-electric effect and find the polarization of spins of the charge carriers in quantizing magnetic field. We show that, in quantizing magnetic field, the oscillations of the spin polarization become “anomalously” large.

EFFECTIVE INTERACTION

The Hamiltonian of the 2D system can be written as:

$$\mathcal{H} = H_k + H_s + H_{ks} + H_{ef} + H_v + H_{ev}. \quad (1)$$

Here H_k and H_s are kinetic and Zeeman energies, respectively, in the magnetic field $\mathbf{H} = (0, 0, H)$. H_{ef} is the

Hamiltonian of the electrons' interaction with the electric field. H_{ev} and H_v are Hamiltonians of the electron-lattice interaction and of the lattice itself, respectively. $H_{ks}(p)$ is the interaction between translational and spin degrees of freedom:

$$H_k = \sum_i \frac{(\mathbf{p}_i - (e/c)\mathbf{A}(\mathbf{x}_i))^2}{2m},$$

$$H_s = \hbar\omega_s \sum_i S_i^z, \quad \hbar\omega_s = g\mu_0 H. \quad (2)$$

S_i^α and p_i^α are operators of the components of the spin and kinetic momentum of the i th electron. g denotes the gyromagnetic ratio of electrons, and μ_0 is Bohr magneton.

$$H_{ef} = -e\mathbf{E} \sum_i \mathbf{r}_i. \quad (3)$$

The most general form of $H_{ks}(\mathbf{p})$ is:

$$H_{ks}(\mathbf{p}) = \sum_j \mathbf{f}(\mathbf{p}_j) \mathbf{S}_j = \sum_{\alpha_1 \alpha_2 \dots \alpha_s; \beta} \phi^{\alpha_1 \alpha_2 \dots \alpha_s; \beta}, \quad (4)$$

$$\phi^{\alpha_1 \alpha_2 \dots \alpha_s; \beta} = \text{const} \sum_j p_j^{\alpha_1} p_j^{\alpha_2} \dots p_j^{\alpha_s} S_j^\beta$$

Here $\mathbf{f}(\mathbf{p}_j)$ is a pseudo-vector, components of which can be represented as a form of order s in the components of the kinetic momentum p_j^α .

Now we perform the canonical transformation of the Hamiltonian. Up to the terms linear in $T(t)$, we have:

$$\tilde{H} = e^{T(p)} \mathcal{H} e^{-T(p)} \approx \mathcal{H} + [T(p), \mathcal{H}]. \quad (5)$$

The operator of the canonical transformation $T(p)$ has to be determined from the requirement that, after the transformation, the k and s subsystems become independent. This requirement can be written as the following condition:

$$H_{ks}(p) + [T(p), H_k + H_s] = 0. \quad (6)$$

Note that, after the canonical transformation, the H_k and H_s operators are the integrals of motion if $H_{ev} = 0$. We assume the specific form of the H_{ks} term, namely, Rashba interaction, which is non-zero even in the linear order in momentum:

$$H_{ks}(p) = \alpha \varepsilon_{zik} \sum_j S_j^i p_j^k = \frac{i\alpha}{2} \sum_j (S_j^+ p_j^- - S_j^- p_j^+), \quad (7)$$

$$S^\pm = S^x \pm iS^y, \quad p^\pm = p^x \pm ip^y.$$

Here α is the constant characterizing the spin-orbit interaction, ε is the fully-antisymmetric Levi—Chivita tensor.

Inserting the operator (7) into the general solution for Eq. (6), we obtain:

$$T(p) = \frac{i\alpha}{2\hbar(\omega_c - \omega_s)} \sum_j (S_j^+ p_j^- - S_j^- p_j^+). \quad (8)$$

The criteria for the applicability of this theory is that, for characteristic values of the electron momentum \bar{p} , the inequality $\alpha\bar{p} \ll \hbar(\omega_c - \omega_s)$ should hold.

Using the explicit form of the operator $T(p)$, we have for the effective interaction:

$$eE^\alpha(t)[x_j^\alpha, T(p)] = -\frac{e\alpha}{2\hbar(\omega_c - \omega_s)} (S^+ E^-(t) + S^- E^+(t)),$$

$$S^\alpha = \sum_i S_i^\alpha. \quad (9)$$

The renormalized interaction with the electric field contains only spin variables, thus, it affects only spin evolution of the conductivity electrons.

Now, taking into account the explicit form of the operator that defines the canonical transformation, we find the operators of power absorbed by the electron subsystem:

$$\begin{aligned} \dot{H}_{e(f)}(t) &= \dot{H}_{k(f)}(t) + \dot{H}_{s(f)}(t) = \frac{e\mathbf{E}(t)\mathbf{p}}{m} + \\ &+ \frac{ie\alpha\omega_s}{2(\omega_c - \omega_s)} \{S^- E^+(t) - S^+ E^-(t)\} = J_e^\alpha E^\alpha(t). \end{aligned} \quad (10)$$

Here

$$\begin{aligned} J_e^\alpha &= eV^\alpha, \\ V^\alpha &= \frac{p^\alpha}{m} + \frac{1}{i\hbar} [x^\alpha, H_{ks}(p)] + \frac{1}{m} [T(p), p^\alpha]. \end{aligned} \quad (11)$$

The operator V^α is the transformed electron velocity in the zeroth order in the field. The expression $J_e^\alpha E^\alpha(t)$ is the operator of power absorbed by both translational and spin degrees of freedom due to the interaction of the conductivity electrons with the electric field. One can write $V^\alpha = V_k^\alpha + V_s^\alpha$, where

$$V_k^\pm = \frac{p^\pm}{m}, \quad V_s^\pm = \mp \frac{i\alpha}{2\hbar(\omega_c - \omega_s)} S^\pm. \quad (12)$$

BALANCE EQUATIONS

Bearing the evolution of the spin subsystem in mind, we write down the balance equations for the Zeeman energy and the transverse components of spin:

$$\begin{aligned} \dot{H}_s &= \frac{\alpha e\omega_s}{2i\hbar(\omega_c - \omega_s)} \{S^+ E^-(t) - S^- E^+(t)\} + \\ &+ \frac{1}{i\hbar} [H_s, \tilde{H}_{ev}] \end{aligned} \quad (13)$$

$$\dot{S}^\pm = \mp i\omega_s S^\pm \mp \frac{ie\alpha}{\hbar(\omega_c - \omega_s)} S^z E^\pm(t) + \frac{1}{i\hbar} [S^\pm, \tilde{H}_{ev}]. \quad (14)$$

We describe the state of the non-equilibrium system with the average values of the following operators: H_k , H_s , N , H_v (N is the operator for number of electrons). In this case, for the non-equilibrium statistical operator (NSO) $\rho(t)$ [7], we have:

$$\rho(t, 0) = \rho_q(t, 0) - \int_{-\infty}^0 dt_1 e^{\varepsilon t_1} \left\{ \int_0^1 d\tau \rho_q^\tau \dot{S}(t + t_1, t_1) \rho_q^{(1-\tau)} + iL_{ef}(t + t_1) \rho_q(t + t_1, t_1) \right\}, \quad (15)$$

$$A(t, t_1) = e^{it_1 L} A(t, 0), \quad iL_{ef} A = (i\hbar)^{-1} [A, H_{ef}].$$

Here $S(t)$ is the entropy operator:

$$S(t) = \Phi(t) + \beta_k(H_k - \mu N) + \beta_s H_s + \beta(H_v + \tilde{H}_{ev}). \quad (16)$$

The parameters β_k , β_s , β have the meaning of the inverse effective temperatures of the kinetic and spin electron subsystems and the inverse equilibrium temperature of the lattice.

$$\dot{S}(t, 0) = \frac{\partial S(t, 0)}{\partial t} + \frac{1}{i\hbar} [S(t, 0), \tilde{H}_{ev}] \quad (17)$$

is the entropy production operator. $\rho_q(t, 0) = \exp\{-S(t)\}$ is the quasi-equilibrium statistical operator.

Averaging the operator equations of motions for the spin subsystem yields:

$$\begin{aligned} \partial_t \langle H_s \rangle &= \frac{\alpha e\omega_s}{2i\hbar(\omega_c - \omega_s)} \{ \langle S^+ \rangle E^-(t) - \langle S^- \rangle E^+(t) \} + \\ &+ \langle \dot{H}_{s(v)} \rangle, \end{aligned} \quad (18)$$

$$\partial_t \langle S^\pm \rangle = \mp i\omega_s \langle S^\pm \rangle \mp \frac{ie\alpha}{\hbar(\omega_c - \omega_s)} \langle S^z \rangle E^\pm(t) + \langle \dot{S}_{(v)}^\pm \rangle. \quad (19)$$

The first term in the right hand side of Eq. (18) represents Joule heat acquired by the spin subsystem of conductivity electrons. The second term describes the relaxation of the longitudinal component of electrons' spin.

Let's turn our mind to the equation (19) describing the evolution of the transverse components of the spin. The collisional term in that formula has the order of $\langle S^\pm \rangle \cdot \nu_2$, where ν_2 is the relaxation frequency of the transverse spin. Below we shall obtain an explicit expression for ν_2 . Now the balance equation for the transverse spin is easily solved, and for the steady-state case we obtain the expression that determines the polarization of electron spins $m^\pm = g\mu \langle S^\pm \rangle$:

$$m^\pm = \pm \frac{i\alpha g\mu e E^\pm}{\hbar(\omega_c - \omega_s)} \cdot \frac{\langle S^z \rangle}{\mp i(\omega - \omega_s) + \nu_2}. \quad (20)$$

$$\langle S^z \rangle = (1/2) \sum_\mu \{f_{\mu\uparrow} - f_{\mu\downarrow}\}, \quad (21)$$

where the symbols \uparrow, \downarrow denote the spin orientation with respect to the z axis.

It follows from Eqs. (20), (21) that the dependence of the average magnetic moment of electrons m^\pm upon the magnetic field is determined by the form of the density of states $\rho(\varepsilon)$. Within the framework of the self-consistent Born approximation, we have [8]:

$$\rho(\varepsilon) = \rho_0 [1 + 2 \sum_{l=1}^{\infty} e^{(-\pi l / \omega_c \tau_0)} \cos(\frac{2\pi l \varepsilon}{\hbar \omega_c})]. \quad (22)$$

Here $\rho_0 = m/\hbar^2\pi$ is the density of states in zero magnetic field, τ_0 is the relaxation time in zero magnetic field. We have omitted the Zeeman splitting from the expression for the density of states. It follows from Eq. (22) that, when $\omega_c \tau_0 \ll 1$, the density of states is $\rho(\varepsilon) = \rho_0$. When the magnetic field is increased, so that $\omega_c \tau \leq 1$, it is sufficient to leave only the first term in the sum. In that case, the oscillating expression for the density of states has the sinusoidal form.

Results of the numerical calculation of the magnetization m^+ are presented on Fig. 1. They were obtained for the following parameters: $m = 0.067m_0$ (m_0 is the free electron mass), the Fermi energy $\mathcal{E}_F = 10$ meV, the 2D electron mobility $\mu \approx 0.1 - 1.0 \times 10^7$ cm²/Vs. It follows from the numerical analysis that the dependence of the electron magnetization upon the magnetic field has oscillating character, and the amplitude of the oscillations is very sensitive to temperature.

Now consider the power absorbed by the spin subsystem. From Eq. (18), we have:

$$Q = \beta_s \left(\frac{\alpha e \omega_s E_\perp}{2\hbar(\omega_c - \omega_s)} \right)^2 2\pi \{G^{+-}(\omega) + G^{-+}(-\omega)\}, \quad (23)$$

where $G^{\pm\mp}(t)$ are Green's functions:

$$G^{\pm\mp}(t) = \theta(-t) e^{\varepsilon t} \langle S^\pm, S^\mp(t) \rangle = \int_{-\infty}^{\infty} d\omega e^{i\omega t} G^{\pm\mp}(\omega),$$

$$(A, B) = \int_0^1 d\tau \text{Sp}(A \rho_q^\tau \Delta B \rho_q^{(1-\tau)}),$$

$$\Delta A = A - \text{Sp}\{A \rho_q\}. \quad (24)$$

$\theta(t)$ is the unit step (Heaviside) function.

Making a chain of motion equations for the Green's function and neglecting terms of more than second order in the interaction H_{ev} , and keeping only zeroth order in thermodynamic forces in terms having the first and the second order in H_{ev} , we obtain:

$$G^{+-}(\omega) = \frac{1}{\pi \beta \hbar \omega_s} \frac{\langle S^z \rangle}{i\omega_s + M^{+-}(\omega)}. \quad (25)$$

The quantity $M^{+-}(\omega)$ is the mass operator calculated in the second order in the electron-lattice interaction and in zeroth order in thermodynamic forces:

$$M^{+-}(\omega) = \frac{\beta \hbar \omega_s}{\langle S^z \rangle} \int_{-\infty}^0 dt e^{\varepsilon t} (\dot{S}_{(v)}^+, \dot{S}_{(v)}^-(t)) \quad (26)$$

The imaginary part of the mass operator $\text{Im } M^{+-}(\omega) = \delta\omega_s$ determines the frequency shift for electron spins, while its real part $\text{Re } M^{+-}(\omega) = \nu_2(\omega)$ plays the role of the inverse relaxation time for the transverse spin components.

The canonical transformation that we performed earlier for decoupling the kinetic and spin subsystems also leads to the renormalization of the electron-lattice interaction Hamiltonian. The effective Hamiltonian of the electron-lattice interaction now becomes $H_{el} + [T, H_{el}]$. Note that the Hamiltonian H_{el} can be presented in the form:

$$H_{el} = H'_{el} + H''_{el},$$

where H'_{el} is the Hamiltonian of the spin-independent part of the electron-lattice interaction (it is responsible, e.g., for the electron momentum relaxation), and H''_{el} is the spin-dependent part, responsible for the relaxation of the electron magnetization. Since the canonical transformation operator $T(p)$ depends upon the electron spin, the Hamiltonian of the total spin-lattice interaction acquires the form:

$$H''_{el} + [T(p), H'_{el}].$$

Here we neglected higher-order terms in the spin-orbit interaction, that arise from the $[T(p), H'_{el}]$ commutator. It can be shown that situations are possible where the contribution of the renormalized part of the conductivity electrons' interaction with the lattice has the same order as the contribution from the ordinary electron-phonon

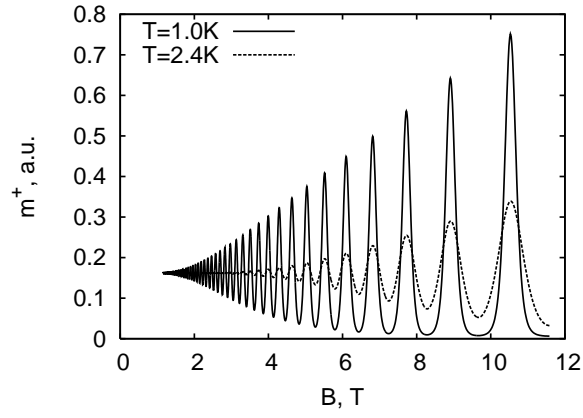


FIG. 1: The magnetization of 2D electron gas vs the magnetic field for different temperatures

interaction, but those two contributions have, obviously,

substantially different dependencies upon the temperature and the magnetic field.

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